JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 22, No. 3, May–June 1999

Two-Body Optimization for Deflecting Earth-Crossing Asteroids

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We present a formulation of a finite-dimensional optimization problem associated with the deflection of Earth-crossing asteroids. The performance measure is minimizing the delta-V requirement for achieving a minimum target separation distance. A number of astrodynamical constraints are identified and modeled. The constrained optimization problem is numerically solved using a sequential quadratic programming method. Our numerical analysis indicates that the minimum delta-V requirement is not a monotonically decreasing function of warning time; rather, there is a finer structure associated with the orbital period of the colliding asteroid. The analysis tool presented here may be used for optimizing the interceptor mission for impact mitigation.

I. Introduction

THE crash of the comet Shoemaker-Levy 9 into Jupiter in July 1994 was a remarkable event. Watching the results[1,2] from the impact, we felt a chilling warning of the possibility of a similar event happening to the Earth.[3] There are at least 1000 Earth-crossing asteroids capable of global environmental catastrophe upon Earth-impact, and five new ones are discovered every year.[4] Crude historical data suggest that we can expect an impact greater than the equivalent of 10 Mtons of TNT about once per century on average.[3] About 50,000 years ago the Barringer crater (1.5 km in diameter) in Arizona was formed by an explosion estimated to be equivalent to about 20 Mtons of TNT.[5] A recent large atmospheric explosion was the great Tunguska bolide (~20 Mtons of TNT) of 1908, which was caused by about a 60-m-diam comet or asteroid and exploded with the force of 1000 Hiroshima bombs. (One Hiroshima bomb has an explosive yield equivalent to about 20 ktons of TNT.)[6]

As a result of the possibility of an asteroid or comet impacting the Earth, several workshops have been held to study the fundamentals of the impact and the impact mitigation problem. The concentration of the workshops has been related primarily on the detection problem, assessing the magnitude of the threat, and impact effects and hazards to Earth, as well as the political implications of developing an impact mitigation capability. Two spacecraft exploration missions, Near Earth Asteroid Rendezvous (NEAR)[7] and Clementine,[8] have included intercepts of asteroids as a major part of their mission to study the nature of asteroids. Despite an increase in interest on the impact mitigation problem,[9–12] little astrodynamical analysis has been performed; in particular, the mathematical optimization problem has received scant attention.[13,14] A number of dynamics and control problems in hazard mitigation are defined in Ref. 14. Here, we further one of these problems in formulating and solving the astrodynamical optimization problem.

The dynamics of the problem is based on a two-dimensional, two-body approximation. The analysis centers on how optimal impulses applied to an Earth-crossing asteroid (ECA) at various points on its orbit affect the outcome when there is a presumption of collision otherwise. The performance measure is minimizing the delta-V required for achieving a minimum target separation distance. The constrained optimization problem is numerically solved using the sequential quadratic programming (SQP) method that is available in the MATLAB™ optimization toolbox.[15] Our numerical analysis indicates that the minimum delta-V requirement is not a monotonically decreasing function of warning time; rather, there is a finer structure associated with the orbital period of the colliding asteroid. We also arrive at some surprising results in the orientation of the optimal direction of the delta-V vector.

Received Jan. 9, 1998; revision received Nov. 12, 1998; accepted for publication Nov. 12, 1998. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

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II. Problem Formulation

Given an ECA and an established Earth-collision, the problem is to minimize the delta-V required to deflect the asteroid in such a manner as to miss the Earth by a minimum target miss distance R_{critical} (see Fig. 1). The performance index is defined by the magnitude of the impulse imparted to the asteroid:

$$J = \sqrt{\Delta v_{\parallel}^2 + \Delta v_{\perp}^2} \tag{1}$$

where $\Delta v_{\parallel}$ and $\Delta v_{\perp}$ are the tangential and normal velocity increments, respectively (see Fig. 1). Obviously, $\Delta v_{\parallel}$ and $\Delta v_{\perp}$ give the magnitude and direction of the impulse. This problem is subject to the two-body equations that are part of the totality of constraints for the optimization problem. The terminal boundary condition at the final time t_f is

$$R - R_{\text{critical}} = 0 \tag{2}$$

where R is the distance between the Earth and asteroid. Strictly speaking, Eq. (2) should be as $R - R_{\text{critical}} \geq 0$. As will be apparent later, this inequality creates an unnecessary extra degree of freedom that causes convergence problems in the numerical optimization scheme; hence, we prefer the equality to the inequality constraint.

Additional constraints to the optimization problem are obtained by noting that after an impulse R is continuous and differentiable. The minimum R (i.e., $R = R_{\text{critical}}$) satisfies the additional necessary conditions[14]:

$$\dot{R} = 0 \tag{3}$$

$$\ddot{R} \geq 0 \tag{4}$$

Equations (2–4) form the totality of constraints for the optimization problem. For these equations to be useful, they must be expressed in terms of the optimization variables such as $\Delta v_{\parallel}$ and $\Delta v_{\perp}$. This functional expression is obtained implicitly from the two-body integrals of motion and the method of Lagrangian coefficients as discussed in the following.

Assuming the ECA orbits the Sun in the ecliptic plane, we have, from the geometry of the problem (see Fig. 1),

$$R = \sqrt{r_{\oplus}^2 + r_a^2 - 2r_{\oplus} r_a \cos(\nu_a - \nu_{\oplus})} \tag{5}$$

where $r_{\oplus}$ is the distance from the Sun to the Earth, r_a is the distance from the Sun to the asteroid, ν_a is the true anomaly of the asteroid, and $\nu_{\oplus}$ is the true anomaly of the Earth. With $D \equiv R^2$, Eqs. (3) and (4) may be written as

$$\dot{R} = \dot{D}/2R = 0 \tag{6}$$

$$\ddot{R} = -(\dot{D}^2/4R^3) + (\ddot{D}/2R) \geq 0 \tag{7}$$

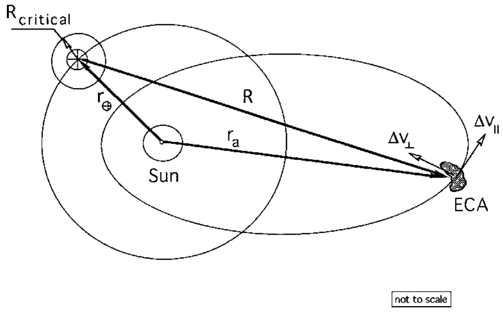

Fig. 1 Geometry of an Earth-crossing asteroid.

where

$$\dot{D} = 2r_\oplus \dot{r}_\oplus + 2r_a\dot{r}_a - 2\dot{r}_\oplus r_a \cos(\nu_a - \nu_\oplus) - 2r_\oplus \dot{r}_a \cos(\nu_a - \nu_\oplus) + 2r_\oplus r_a \sin(\nu_a - \nu_\oplus)(\dot{\nu}_a - \dot{\nu}_\oplus) \tag{8}$$

$$\ddot{D} = 2\Big[\dot{r}_\oplus^2 + r_\oplus\ddot{r}_\oplus + \dot{r}_a^2 + r_a\ddot{r}_a - \cos(\nu_a - \nu_\oplus) \times (r_\oplus\ddot{r}_a + 2\dot{r}_\oplus\dot{r}_a + \ddot{r}_\oplus r_a) + 2\sin(\nu_a - \nu_\oplus)(\dot{\nu}_a - \dot{\nu}_\oplus) \times (\dot{r}_a r_\oplus + r_a\dot{r}_\oplus) + r_\oplus r_a \cos(\nu_a - \nu_\oplus)(\dot{\nu}_a - \dot{\nu}_\oplus)^2 + r_\oplus r_a \sin(\nu_a - \nu_\oplus)(\ddot{\nu}_a - \ddot{\nu}_\oplus)\Big] \tag{9}$$

The time rates of change of the various quantities in the preceding equations can be obtained from the two-body equations:

$$\dot{r} = \frac{\mu e \sin\nu}{\sqrt{\mu a(1-e^2)}} \tag{10}$$

$$\dot{\nu} = \frac{\sqrt{\mu a(1-e^2)}}{r^2} \tag{11}$$

$$\ddot{r} = r\dot{\nu}^2 - (\mu/r^2) \tag{12}$$

$$\ddot{\nu} = -(2/r)\dot{r}\dot{\nu} \tag{13}$$

where e and a are the eccentricity and semimajor axis of the asteroid, and μ is the Sun's gravitational constant. From the method of Lagrangian coefficients,[16] the position and velocity vectors $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$ can be described in terms of initial conditions $\boldsymbol{r_0}(t_0)$ and $\boldsymbol{v_0}(t_0)$. Immediately after the application of the impulse, the orbit of the asteroid is perturbed by a delta-V; hence, using $t_0 = t_{\text{impulse}}$, we can write

$$\boldsymbol{r}_0(t_{\text{impulse}}) = \boldsymbol{r}_i \tag{14}$$

$$\boldsymbol{v}_0(t_{\text{impulse}}) = \boldsymbol{v}_i + \Delta\boldsymbol{v} \tag{15}$$

where $\boldsymbol{r}_i$ and $\boldsymbol{v}_i$ are the original position and velocity of the asteroid just before the impulse. From $\boldsymbol{r}_0(t_{\text{impulse}})$ and $\boldsymbol{v}_0(t_{\text{impulse}})$, $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$ at time t may be calculated from

$$\boldsymbol{r}(t) = F\boldsymbol{r}_0 + G\boldsymbol{v}_0 \tag{16}$$

$$\boldsymbol{v}(t) = F_t\boldsymbol{r}_0 + G_t\boldsymbol{v}_0 \tag{17}$$

where the Lagrangian coefficients are given by

$$F = 1 - (a/r_0)(1 - \cos\theta) \tag{18}$$

$$G = (t - t_0) + \left(a\sqrt{a}/\sqrt{\mu}\right)(\sin\theta - \theta) \tag{19}$$

$$F_t = -\left(\sqrt{\mu a}/rr_0\right)\sin\theta \tag{20}$$

$$G_t = 1 - (a/r)(1 - \cos\theta) \tag{21}$$

where

$$\theta \equiv E - E_0 \tag{22}$$

$$r = a(1 - e\cos E) \tag{23}$$

In the preceding equations the perturbed orbital elements of the asteroid are calculated from $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$. The eccentric anomaly E at t is obtained by solving Kepler's time equation.

Defining the optimization vector $\boldsymbol{x}$ as the set of free parameters $\boldsymbol{x} = [\Delta v_\parallel, \Delta v_\perp, t_f]$, the nonlinear programming problem (NLP) can be formulated as follows. Determine $\boldsymbol{x}$ that minimizes the objective function $J(\boldsymbol{x})$ given by Eq. (1), while satisfying the set of equality and inequality constraints given by Eqs. (2–4). Note that the parameter t_f is a free optimization variable that describes the time at which the constraints given by Eqs. (2–4) are satisfied.

III. Preliminary Analysis

Before solving the optimization problem numerically, it is useful to derive some results that will aid us in the analysis to follow. Suppose a small arbitrary impulse is applied to an asteroid at an arbitrary point on its orbit. The orbital elements are perturbed by[17]

$$\Delta a = \frac{2}{n\sqrt{1-e^2}}[e\sin\nu\Delta v_s + (1 + e\cos\nu)\Delta v_t] \tag{24}$$

$$\Delta e = \frac{\sqrt{1-e^2}}{na}\left[\sin\nu\Delta v_s + \left(\frac{e+\cos\nu}{1+e\cos\nu} + \cos\nu\right)\Delta v_t\right] \tag{25}$$

$$\Delta\omega = \frac{\sqrt{1-e^2}}{nae}\left[-\cos\nu\Delta v_s + \left(1 + \frac{1}{1+\cos\nu}\right)\sin\nu\Delta v_t\right] \tag{26}$$

where Δv_s and Δv_t are the velocity increments along and at right angles to the radius vector (see Fig. 2) and Δa, Δe, and $\Delta\omega$ are the changes in the semimajor axis, the eccentricity and the longitude of perihelion, respectively, and n is the mean motion. From elementary geometry we have (see Fig. 2)

$$\Delta v_t = \sin\phi\Delta v_\parallel - \cos\phi\Delta v_\perp \tag{27}$$

$$\Delta v_s = -\cos\phi\Delta v_\parallel - \sin\phi\Delta v_\perp \tag{28}$$

where ϕ is the angle subtended by the radius and velocity vectors at the asteroid. The angle ϕ satisfies the following identities[17]:

$$\sin\phi = \frac{1 + e\cos\nu}{(1 + e^2 + 2e\cos\nu)^{\frac{1}{2}}}$$

or

$$\cos\phi = -\frac{e\sin\nu}{(1 + e^2 + 2e\cos\nu)^{\frac{1}{2}}} \tag{29}$$

Substituting Eqs. (27) and (28) into Eqs. (24–26) yields

$$\Delta a = \frac{2}{n\sqrt{1-e^2}}\{[-e\sin\nu\cos\phi + (1 + e\cos\nu)\sin\phi]\Delta v_\parallel - [e\sin\nu\sin\phi + (1 + e\cos\nu)\cos\phi]\Delta v_\perp\} \tag{30}$$

$$\Delta e = \frac{\sqrt{1-e^2}}{na}\left\{\left[\frac{(e + 2\cos\nu + e\cos^2\nu)\sin\phi}{1 + e\cos\nu} - \sin\nu\cos\phi\right] \times \Delta v_\parallel - \left[\frac{(e + 2\cos\nu + e\cos^2\nu)\cos\phi}{1 + e\cos\nu} + \sin\nu\sin\phi\right]\Delta v_\perp\right\} \tag{31}$$

$$\Delta\omega = \frac{\sqrt{1-e^2}}{nae}\left\{\left[\cos\nu\cos\phi + \left(\frac{2+\cos\nu}{1+\cos\nu}\right)\sin\nu\sin\phi\right]\Delta v_\parallel + \left[\cos\nu\sin\phi - \left(\frac{2+\cos\nu}{1+\cos\nu}\right)\sin\nu\cos\phi\right]\Delta v_\perp\right\} \tag{32}$$

It is straightforward to show that the term associated with $\Delta v_\perp$ in Eq. (30) simplifies to

$$e\sin\nu\sin\phi + (1 + e\cos\nu)\cos\phi = \frac{(1 + e^2 + 2e\cos\nu)^{\frac{1}{2}}}{2v^2}\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \tag{33}$$

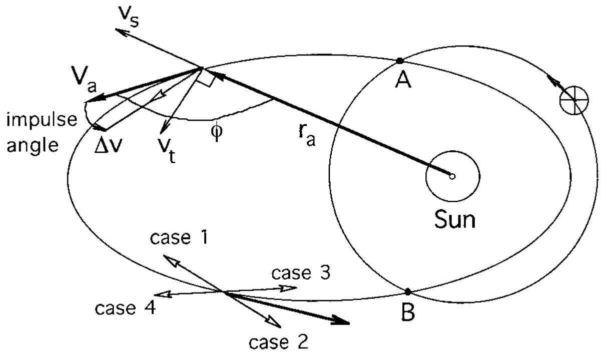

Fig. 2 Description of some astrodynamical quantities.

where H is the angular momentum of the asteroid. Thus, we have

$$\Delta a = \frac{2}{n\sqrt{1-e^2}}\{[-e\sin\nu\cos\phi + (1+e\cos\nu)\sin\phi]\Delta v_{\parallel}\} \quad (34)$$

Because the change in the semimajor axis due to $\Delta v_{\perp}$ is zero, the orbital period is unchanged by $\Delta v_{\perp}$. Hence, if an impulse perpendicular to the velocity vector is applied to the asteroid, it returns to the point of application of Δv at exactly the same time as the unperturbed asteroid after multiple periods. Hence, at exact multiple periods of the asteroid before an impact, the optimal impulse should be parallel to the velocity vector. Thus, for example, if the impact occurs at point A (see Fig. 2), then the optimal delta-V vector is parallel to the velocity vector at point A at exact multiples of the asteroid's period. This result will be used as one of the key tests in validating the numerical analysis.

IV. Numerical Analysis

Procedure

The NLP formulated in Sec. II was solved by using the MATLAB optimization tool box. For well-documented reasons the heliocentric canonical units were used. In this system the distance units (DU), time units (TU), and speed units (SU) are 1 DU = 1 AU = 1.49596 E8 km, 1 TU = $1/2\pi$ year = 58.17 days, 1 SU = 29.80 km/s.

In the discussions to follow, we will use the term Impulse Time to specifically mean either ($t_{\text{impact}} - t_{\text{impulse}}$) or its absolute value—the exact usage will be obvious from the context. Here t_{impact} denotes the time at collision. Although the impact time is quite close to t_f, it is not the same because t_f is the time when $R = R_{\text{critical}}$ (see Sec. II). Also, because t_{impact} and t_{impulse} are not independent quantities, we choose $t_{\text{impact}} = 0$. This initialization has the advantage of interpreting t_{impulse} as the time interval prior to impact if no action (i.e., delta-V maneuver) is undertaken. Also, it is apparent that we must have a warning time (i.e., the time interval between detection and collision) greater than the impulse time. Because the impulse time provides a lower limit for the warning time, we will sometimes use impulse times and warning times interchangeably.

To compare our results to prior work,[9,10,12] we initially used $R_{\text{critical}} = 1$ Earth radius (4.263E-5 DU). For this value of R_{critical}, the minimum ΔV is so small that the time slope of R near R_{critical} is quite sharp so that it created some convergence problems with the code. A larger R_{critical} created a shallower slope for $R(t)$ and hence a faster convergence. By changing R_{critical} to 10 Earth radii, the convergence problem was completely eliminated, and the convergence was faster than that for R_{critical} less than 10 Earth radius. The delta-V requirement varies linearly with R_{critical} so long as the applied ΔV is small,[9] and our numerical results show the same phenomena. Hence, although the code was used for $R_{\text{critical}} = 10$ Earth Radii, the results are displayed for $R_{\text{critical}} = 1$ Earth radius by dividing the delta-V by 10.

A second issue to note in the numerical solution of the optimization problem is the sensitivity of the optimization algorithm to initial guesses. Regardless of the value of R_{critical}, if the initial guess is not a good one, the code does not converge. To generate good guesses for a first use of the code, the following procedure was adopted. The impulse vector $[\Delta v_{\parallel}, \Delta v_{\perp}]$ is guessed to be zero for an impulse time of 1TU (i.e., $t_{\text{impact}} - t_{\text{impulse}} = -1$TU). The unconverged

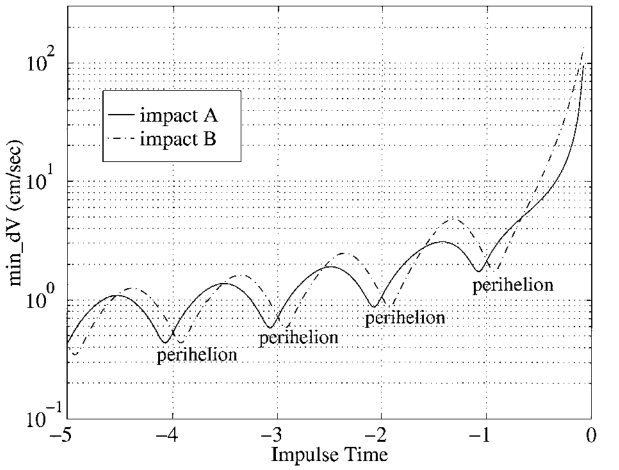

Fig. 3 Minimum Δv for impact scenarios A and B (Abscissa is in fractions of the orbital period.).

delta-V from the code is used as an input for an independent new code that propagates $R(t)$ as described in Sec. II. The minimum miss distance can be obtained easily by plotting $R(t)$ in the vicinity of $t = 0$. The impulse vector not being the optimal solution, the minimum of $R(t)$ is not equal to R_{critical}. If the minimum R is less (more) than R_{critical}, then an increased (decreased) magnitude of the impulse vector is used as a guess for the optimization algorithm and the process repeated until the delta-V converges. As a double-check, the converged impulse vector is used to propagate $R(t)$ to ensure that the minimum R is indeed equal to R_{critical} (typically, min R and R_{critical} are within 0.01% of each other). The 1TU optimal parameters serve as excellent guesses for incremental impulse times, and the optimization algorithm now essentially works in a turnkey fashion.

Results

Consider a fictitious ECA whose orbital elements are given by $a = 1.5$ AU, and $e = 0.5$. Figure 3 shows the magnitude of the minimum impulse as a function of the impulse time for a requirement of R_{critical} equal to 1 Earth radius. The impulse time is normalized to the period of the unperturbed asteroid for ease of interpretation. For this example one period of the asteroid is 1.85 years. The minimum impulse requirements for impact scenarios A and B (see Fig. 2) are expectedly different, but similar. The differences are due to the different geometric positions of the impulse point with respect to the Sun. It is clear from the figure that for impulse times less than about one orbital period the minimum required impulse increases quite dramatically and secularly for decreasing impulse times. For warning times greater than one period of the asteroid, the required minimum delta-V has a cyclic component imposed upon a secular variation. As expected, the secular variation varies inversely with the impulse time and may be considered as a crude first-order approximation to the minimum required delta-V. The period of the cyclic component is equal to the period of the asteroid. As discussed in Sec. III, this is to be expected because the perturbation in the orbital elements is periodic with respect to the true anomaly. The local minima in Fig. 3 represent the perihelion of the asteroid, but the local maxima do not represent the aphelion.

The components of the impulse vector for scenario A are shown in Fig. 4. Notice that for impulse times beyond about 0.75 (of the orbital period) the contribution to the total delta-V from the perpendicular component is so small that it is barely distinguishable in the graph. To better explain the optimal impulse vector, we define the impulse angle as the angle from the original velocity vector to the impulse vector through the sun-ward direction (see Fig. 2). The optimal impulse angle is displayed in Fig. 5. It is apparent from this figure that for very short warning times the impulse angle converges to an angle perpendicular to the initial velocity. Intuitively, this is reasonable if a rectilinear motion is assumed for short warning times. Further, notice that at exact integral periods of the impulse time, the optimal impulse is antiparallel to the velocity vector. This is in total agreement with the analytical results of Sec. III. The somewhat

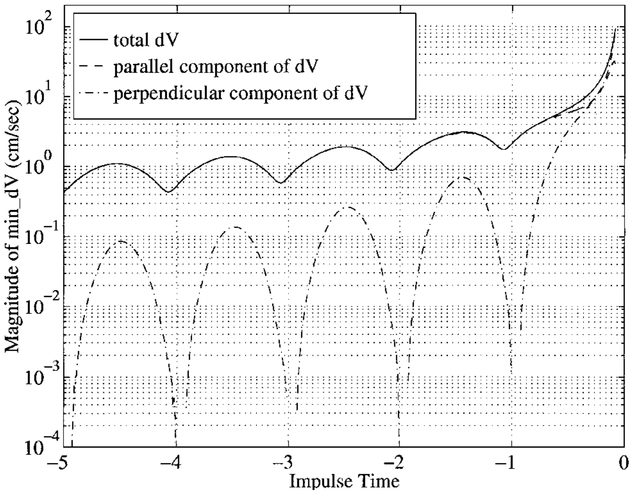

Fig. 4 Components of the minimum Δv.

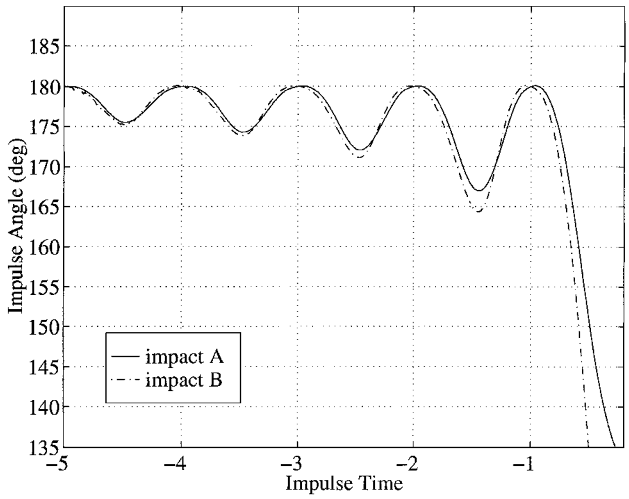

Fig. 5 Optimal impulse angle for impact scenarios A and B.

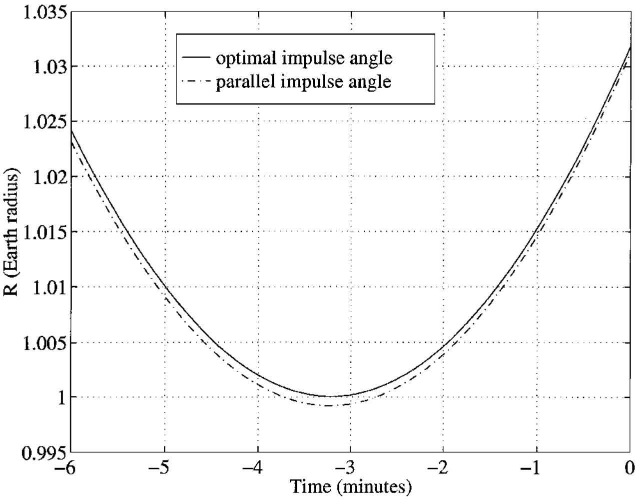

Fig. 6 Optimal versus parallel impulse at perihelion.

surprising result here is that the optimal impulse angle is neither 0° nor 180° at both the perihelion and aphelion impulse points (cf. Figs. 3 and 5). This contradicts the assumptions in Refs. 8–10. A better demonstration of this phenomenon is obtained when $a = 1.1$ and $e = 0.3$. For this example, the optimal impulse angle at the first perihelion point was determined to be 177.6739°. The corresponding minimum delta-V is equal to 4.04175 cm/s. The correctness of this result (i.e., nonparallel optimal angle) is demonstrated in Fig. 6. This figure displays $R(t)$ in the neighborhood of

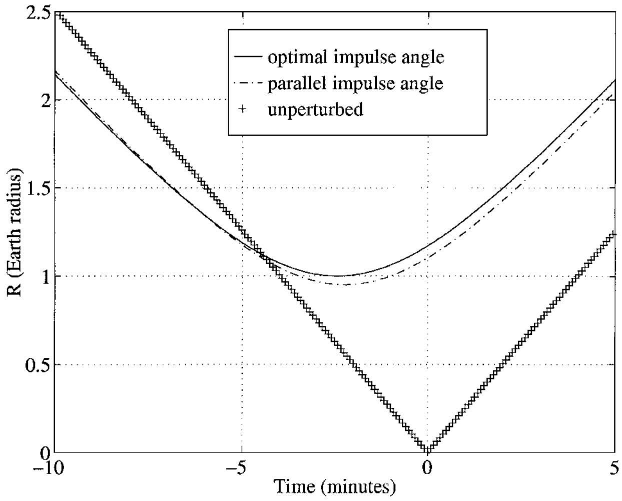

Fig. 7 Optimal versus parallel impulse at aphelion.

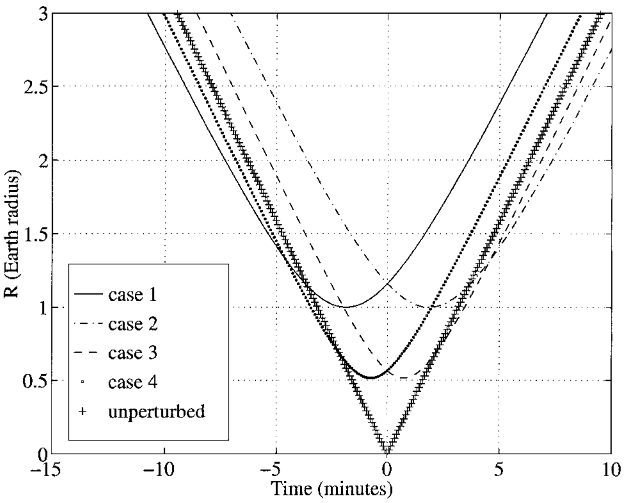

Fig. 8 Two optimal solutions: case 1 and case 2.

the closest approach for two cases: one for the orbit perturbed optimally according to the numerical result (i.e., $\Delta v = 4.04175$ cm/s and impulse angle = 177.6739°) and another for an impulse angle of 180° and $\Delta v = 4.04175$ cm/s. The parallel impulse yields a miss-distance clearly less than R_{critical} (by about 0.08% or equivalently by 5 km) and hence not optimal, whereas the optimal impulse produces a minimum miss distance of exactly 1 Earth Radius. Although the differences are apparently small, they may be used effectively for optimizing future interceptor missions. A corresponding plot for the aphelion impulse point for this example generated similar results, but the differences were smaller for this particular example. A better demonstration of this phenomenon is obtained for $a = 2.0$ and $e = 0.8$. The numerically optimal impulse for this example for the first aphelion point is a Δv of 2.64254 cm/s and an impulse angle of 161.8875°. Figure 7 shows results similar to Fig. 6: the miss distance is 1 Earth radius for the optimal values while a parallel impulse produces a significantly lower miss distance (by about 5% or equivalently by about 316 km). The differences are much larger here (than the previous example) because of the increased difference between the optimal impulse angle and 180°. Included in this figure is a plot of the unperturbed orbit that demonstrates imminent collision. To varying orders of magnitude, this same phenomenon was observed for different orbital elements of the asteroid.

For any given impulse time the problem has two solutions for the optimal impulse angle separated by 180°. These two solutions are depicted in Fig. 2 as cases 1 and 2, which yield the miss distance equal to 1 Earth radius. An example of this is provided in Fig. 8 for an arbitrary impulse time (= −1.5 period) for an asteroid with $a = 1.75$ and $e = 0.9$. Also shown in this figure is the solution to cases 3 and 4, which do not provide the miss distance equal to 1 Earth radius

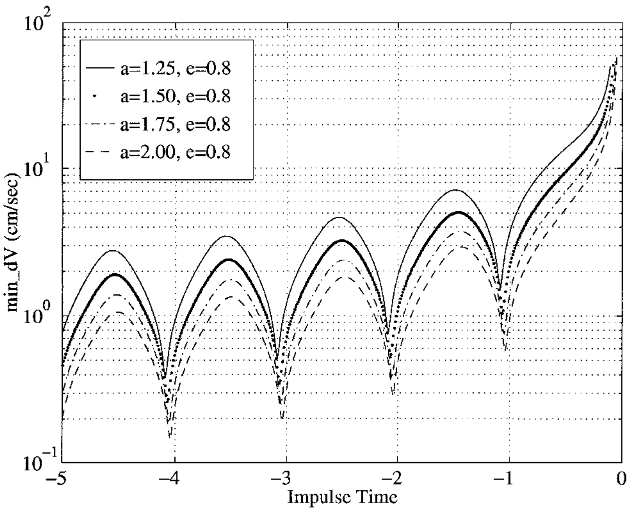

Fig. 9 Minimum Δ v for varying semimajor axes.

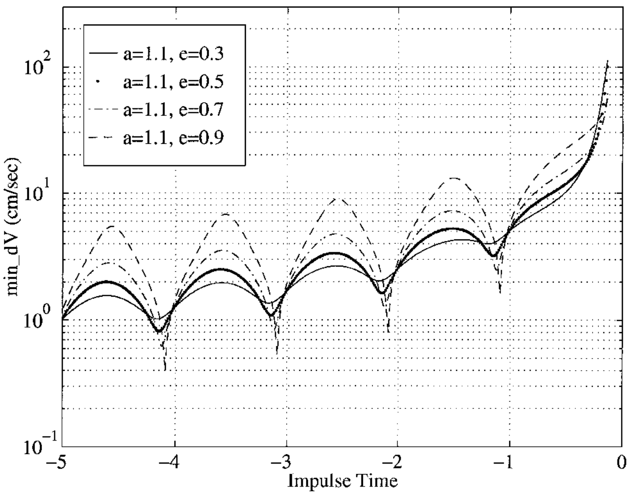

Fig. 10 Minimum Δ v for varying eccentricities.

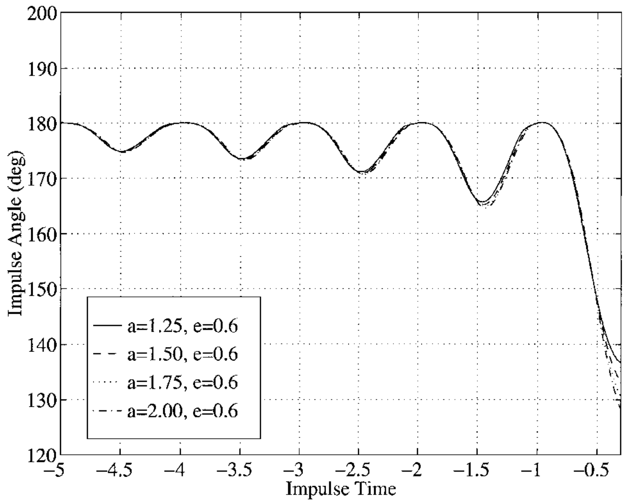

Fig. 11 Optimal impulse angles for varying semimajor axes.

with the same delta-V as that of cases 1 and 2. These cases (see Fig. 2) are at exact opposite angles from the optimal solution and the orbit tangent.

Finally, Figs. 9 and 10 demonstrate that the same trends are apparent for asteroids with different orbital elements. In Fig. 9 the eccentricity is fixed at $e = 0.8$, and the semimajor axis is varied from 1.25 AU to 2.0 AU in steps of 0.25 AU. The abscissa represents the impulse time normalized to the period of the corresponding

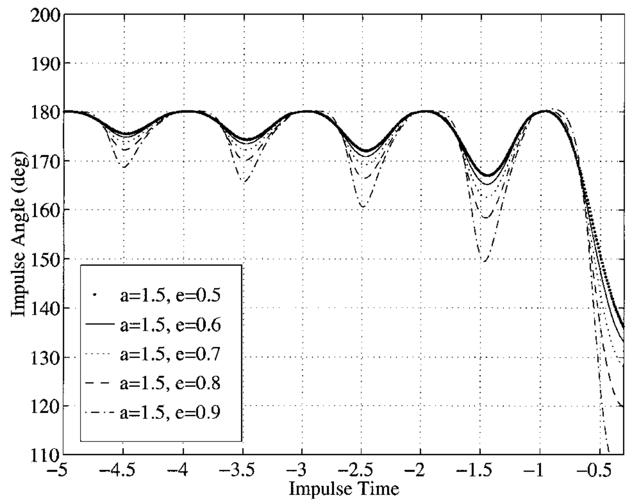

Fig. 12 Optimal impulse angles for varying eccentricities.

semimajor axis. The minimum delta-V decreases with increasing semimajor axis because the orbit with a larger semimajor axis has a longer orbital period (in real time), and hence a longer warning time. In Fig. 10 the semimajor axis is fixed at $a = 1.1$, and the eccentricity is varied from 0.3 to 0.9 in steps of 0.2. It is apparent from this figure that the amplitude of the cyclic component of the delta-V fluctuates with an increasing amplitude for increasing eccentricity. The optimal impulse angles for varying semimajor axes and eccentricities are shown in Figs. 11 and 12. The phenomena discussed in the preceding paragraphs are apparent in these figures as well.

V. Conclusions

The problem of optimizing the impulse required for deflecting ECAs can be cast in terms of a standard NLP problem. The optimal delta-V and impulse angles can be obtained numerically as a function of the impulse time. Even the two-body problem is quite involved, and the results exhibit some nonintuitive phenomena. The miss distance achieved by an impulse is strongly dependent on the location of the impulse on the orbit as well as the direction of the impulse with respect to the orbital velocity. Numerical solutions to this NLP problem for a class of ECAs demonstrate that there are at least two time scales of importance: a first-order time constant associated with the impulse time and a periodic second-order effect associated with the period of the asteroid. Thus, the optimal time for the application of the delta-V is the earliest possible perihelion for warning times greater than one orbital period. This optimal delta-V is not quite parallel to the velocity vector, and this deviation from the parallel could be used for achieving a better performance from a future interceptor mission.

Acknowledgments

Funding for this research was provided, in part, by the U.S. Air Force Space Command and the Naval Postgraduate School. We would like to express our special gratitude to LCDR Brian D. Neuenfeldt at the U.S. Air Force Space Command for sponsoring this research. This work was performed while the first author held a National Research Council-Naval Postgraduate School Research Associateship.

References

[1]Shoemaker, E. M., "Comet Shoemaker-Levy 9 at Jupiter," *Geophysical Research Letter*, Vol. 22, No. 12, 1995, p. 1555.

[2]Levy, D. H., Shoemaker, E. M., and Shoemaker, C. S., "Comet Shoemaker-Levy 9 Meets Jupiter," *Scientific American*, Vol. 273, Aug. 1995, pp. 85–91.

[3]Morrison, D., "Target: Earth!," *Astronomy*, Vol. 23, Oct. 1995, pp. 34–41.

[4]Morrison, D., Chapman, C. R., and Slovic, P., "The Impact Hazard," *Hazards Due to Comets and Asteroids*, edited by T. Gehrels, Univ. of Arizona Press, Tucson, AZ, 1994, pp. 59–91.

[5]Grieve, R. A., and Shoemaker, E. M., "The Record of Past Impacts on Earth," *Hazards Due to Comets and Asteroids*, edited by T. Gehrels, Univ. of Arizona Press, Tucson, AZ, 1994, pp. 417–462.

[6]Stone, R., "The Last Great Impact on Earth," *Discover*, Vol. 17, Sept. 1996, pp. 60–71.

[7]Farquhar, R. W. (ed.), Special Issue on the Near-Earth Asteroid Rendezvous Mission, *Journal of the Astronautical Sciences*, Vol. 43, No. 4, 1995.

[8]Rustan, P. L. (ed.), Special Section: Clementine Mission, *Journal of Spacecraft and Rockets*, Vol. 32, No. 6, 1995, pp. 1044–1078.

[9]Ahrens, T. J., and Harris, A. W., "Deflection and Fragmentation of Near-Earth Asteroids," *Nature*, Vol. 360, Dec. 1992, pp. 429–433.

[10]Solem, J. C., "Interception of Comets and Asteroids on Collision Course with Earth," *Journal of Spacecraft and Rockets*, Vol. 30, No. 2, 1993, pp. 222–228.

[11]Solem, J. C., "Nuclear Explosive Propelled Interceptor for Deflecting Objects on Collision Course with Earth," *Journal of Spacecraft and Rockets*, Vol. 31, No. 4, 1994, pp. 707–709.

[12]Ivashkin, V. V., and Smirnov, V. V., "An Analysis of Some Methods of Asteroid Hazard Mitigation for the Earth," *Planetary and Space Science*, Vol. 43, No. 6, 1995, pp. 821–825.

[13]Conway, B. A., "Optimal Low-Thrust Interception of Earth-Crossing Asteroids," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 5, 1997, pp. 995–1002.

[14]Hall, C. D., and Ross, I. M., "Dynamics and Control Problems in the Deflection of Near-Earth Objects," *Advances in the Astronautical Sciences, Astrodynamics 1997*, Vol. 97, Pt. I, 1997, pp. 613–631.

[15]Grace, A., *Optimization Toolbox User's Guide*, MathWorks, Natick, MA, 1992.

[16]Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, New York, 1987, Chap. 3.

[17]Roy, A. E., *Orbital Motion*, 3rd ed., Inst. of Physics Publishing, Philadelphia, 1988, Chaps. 2–5.